One way to name the intervals of any EDO from 5 to 72 (with 22edo as an example)

Introduction

When extending the familiar diatonic interval names to equal divisions of the octave having more or less than 12 notes, and having sizes of fifth outside the range of ¼-comma meantone to 12-equal, it is unavoidable that we will break some of the familiar interval stacking relationships or that we will break the familiar relationship between how dyads (and hence chords) are named and how they sound.

One can use interval names based on non-diatonic scales such as the Moment-Of-Symmetry (MOS) scales of the various linear temperaments available within a given EDO. These may have more or less than 7 nominals. In general, such names depart radically from the familiar. The system of this paper avoids revolution in favour of evolution.

It is possible to construct an interval naming system that has a one-to-one correspondence with the accidentals of a chain-of-fifths based pitch notation such as Sagittal notation, but this will completely break the relationship of name to sound. It is also possible to construct an interval naming system based on just intonation, but this will completely break the familiar interval stacking relationships and their symmetries, when cast into most EDOs.

The system described in this paper steers a middle course. It preserves the approximate size of the named intervals (to within about +-25 cents) and hence preserves the approximate sound of the named dyads, while also guaranteeing to preserve the symmetries embodied in the following interval stacking relationships (and their octave inversions and extensions). Among other things, they guarantee symmetry within each disjunct tetrachord as well as within the octave:

m3 + M2 = P4
M3 + m2 = P4
M3 + m3 = P5
P4 + m3 = m6
P5 + m2 = m6
P4 + M3 = M6
P5 + M2 = M6
P5 + m3 = m7
P5 + M3 = M7
d5 + A4 = P8
P5 + P4 = P8
m6 + M3 = P8
M6 + m3 = P8
m7 + M2 = P8
M7 + m2 = P8

It does *not* guarantee to preserve the following relationships, except within the ¼-comma meantone to 12-equal range of fifth sizes.

M2 + m2 = m3
M2 + M2 = M3
M3 + M2 = A4
m3 + m3 = d5
P4 + m2 = d5
P4 + M2 = P5
A4 + m2 = P5
d5 + M2 = m6
A4 + m3 = M6
P4 + P4 = m7
d5 + M3 = m7
m6 + M2 = m7
M6 + m2 = m7
A4 + P4 = M7
M6 + M2 = M7

The names

To the conventional interval qualifiers: perfect, minor/major, diminished/augmented, we add the new qualifiers: neutral, sub/super and narrow/wide. Narrow and wide are only needed for EDOs with many notes (> 34). Neutral is midway between major and minor. The terms neutral, sub and super have been in use in microtonal circles since at least the 1960's thanks to Adriaan Fokker, with the same meaning they have here. I think this use of "neutral" dates back to at least Helmholtz and Ellis in 1877.

I use the following abbreviations in this paper. When writing chord names it may be wise to use the longer "dim" and "Aug" to avoid confusing d and A with note names.

Abbreviations				Relative size	
N	Neutral	P	Perfect	0	
n	narrow	W	Wide	1	(Not used for EDOs <= 34)
m	minor	M	Major	2	
s	sub	S	Super	3	
d	diminished	A	Augmented	5	

I note that "small" and "Large" may be substituted for "narrow" and "Wide", but then care must be taken with abbreviations, since "s" is already used for "sub". Perhaps use "l" since it is the last letter of "small" and the first letter of "little".

The method

To determine the names for the intervals of an EDO having *e* divisions to the octave, we first calculate the number of degrees in the EDO's best fifth as

f = Round(*e* * 702 / 1200), where “Round()” means “round to the nearest whole number”.

For example, in 22edo, *e* = 22 and *f* = 13.

The points of reference for the seven interval name classes (unisons through sevenths) are the Perfect and Neutral intervals, arranged in two chains of the EDO’s best fifths, offset by half a fifth as follows.

	P4	---	P1	---	P5	
N2	---	N6	---	N3	---	N7

If *f* is an odd number, the EDO will not *contain* any neutral intervals, but the neutrals will still be used as points of reference, midway between two notes, for naming the other intervals.

We then draw a circle representing an octave, with marks around its circumference representing the notes of the EDO. You can see such a diagram about halfway through this document.

We mark P1 at the top, count *f* divisions clockwise and mark P5.
From P1, we count *f* divisions anticlockwise and mark P4.
From P1, we count *f*/2 divisions clockwise and mark N3.
From P5, we count *f*/2 divisions clockwise and mark N7.
From P1, we count *f*/2 divisions anticlockwise and mark N6.
From P4, we count *f*/2 divisions anticlockwise and mark N2.

For 22edo, *f*/2 = 6.5.

We then use the table below to choose qualifiers corresponding to various numbers of degrees of the EDO, on either side of the perfect and neutral intervals, so as to fill in the gaps, and further to provide alternative names for most intervals.

For EDOs 34 and lower, it should be possible to choose the names from those which are highlighted and do not involve the terms narrow or Wide. To name the intervals of 72edo we need to use *all* of the qualifiers shown in the diagram. For other EDOs we need to determine, for each qualifier, which degree (or half-degree) of the EDO it is closest to, and could therefore be validly used for. To do this we apply the following formula to each degree of 72edo (d_{72}).

d_e = Round(d_{72} * e * 2 / 72) / 2

The results of this for 22edo are shown in the rightmost column.

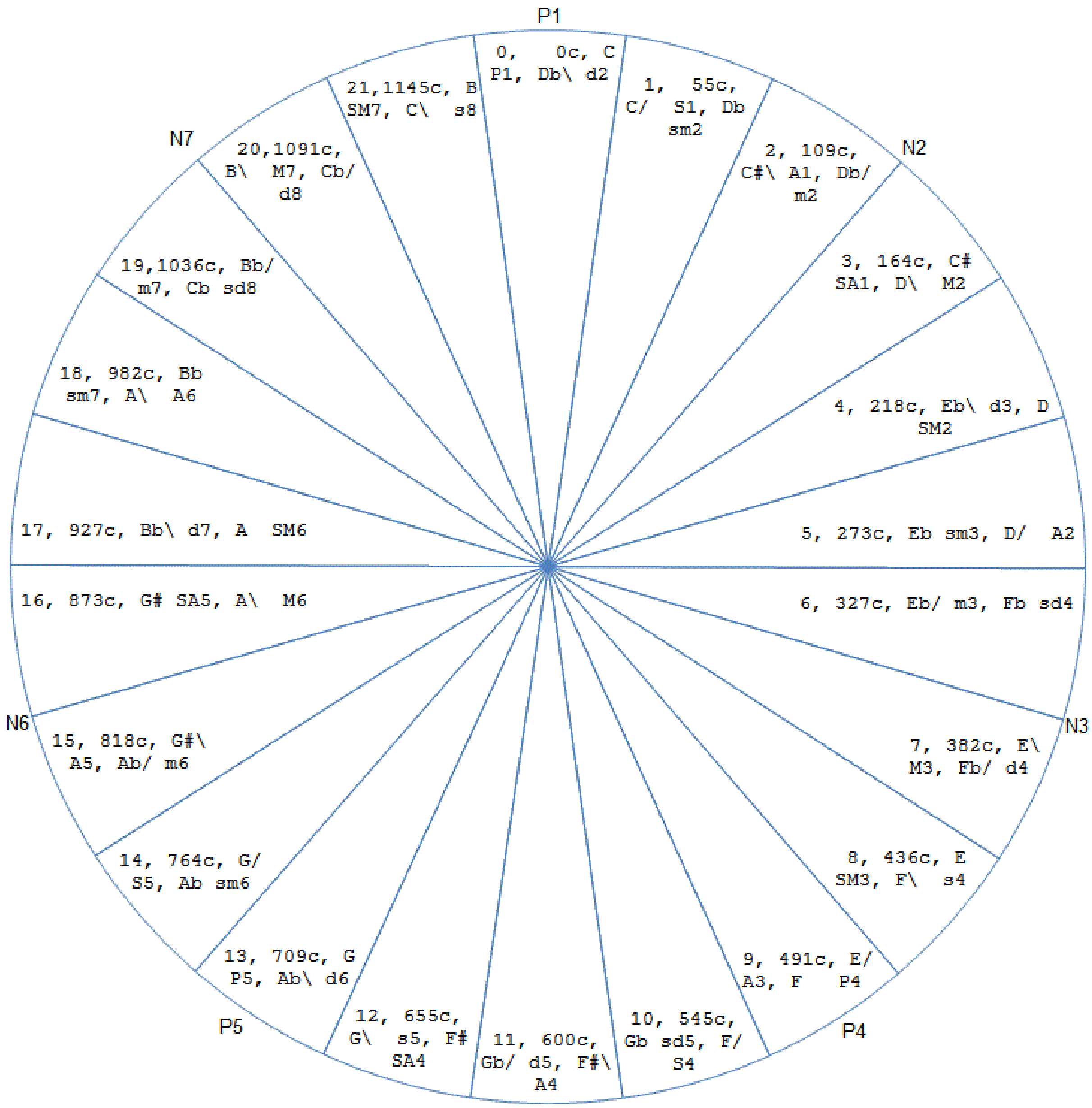
		unisons				seconds				
Degrees		fourths				thirds			Degrees	
of		fifths				sixths			of	
72edo		octaves				sevenths			72edo	22edo
12				WWAA			AA		12	3.5
11				WAA				WSA	11	3.5
10			AA				SA		10	3
9				WSA				nSA	9	3
8			SA				WA		8	2.5
7				nSA		A			7	2
6			WA				WSM		6	2
5		A				SM			5	1.5
4			WS				nSM		4	1
3		S				WM			3	1
2			nS		M				2	0.5
1		W				WN			1	0.5
0	P				N				0	0
-1		n				nN			-1	-0.5
-2			Ws		m				-2	-0.5
-3		s				nm			-3	-1
-4			ns				Wsm		-4	-1
-5		d				sm			-5	-1.5
-6			nd				nsm		-6	-2
-7				Wsd		d			-7	-2
-8			sd				nd		-8	-2.5
-9				nsd				Wsd	-9	-3
-10			dd				sd		-10	-3
-11				ndd				nsd	-11	-3.5
-12				nndd			dd		-12	-3.5

If the fifth-size *f* is an even number, we use the whole numbers of degrees to look up the qualifiers for both the perfect and the neutral intervals. If the fifth size is odd we use the whole numbers only for the perfect intervals, and we use the half numbers for the neutral intervals. When more than one qualifier is valid, we choose the simplest set for which the parts add logically. For example, if S (super) is 1 degree and A (Augmented) is 2 then SA (superaugmented) should be 3. Be aware that in the case of the neutral intervals, for the purpose of such arithmetic, Augmented is really Augmented Major (AM) and diminished is really diminished minor (dm), but the "Major" and "minor" parts are dropped by long-standing convention. If such a set contains qualifiers beginning with n for narrow or W for Wide, but dropping these parts would cause no ambiguity, then they should be dropped. This appears to be possible for EDOs 34 or less.

For the Perfect intervals in 22edo we initially obtain the sequence P S WA WSA (0 1 2 3) and we drop the W's to obtain P S A SA (0 1 2 3). The full sequence for the Perfect intervals is then sd d s P S A SA (-3 -2 -1 0 1 2 3). So we label the intervals either side of P4 as sd4 d4 s4 P4 S4 A4 SA4, and similarly for P5 and P1 (which is also P8). See the circular diagram below.

For the Neutrals we look at the half degrees and initially obtain the sequence (N) M SM WA WSA (0 0.5 1.5 2.5 3.5). I put N in parenthesis here because it is not an interval of the tuning, but merely a reference point. We drop the W's to obtain (N) M SM A SA. Note that, for the purpose of the degree arithmetic, this is effectively (N) M SM AM SAM. The full sequence for the Neutral intervals is then sd d sm m (N) M SM A SA (-3.5 -2.5 -1.5 -0.5 0 0.5 1.5 2.5 3.5). So we label the intervals either side of N3 as sd3 d3 sm3 m3 (N3) M3 SM3 A3 SA3, and similarly for N2, N6 and N7.

The diagram below gives the number of 22edo degrees and the number of cents in each interval, and two names for each interval, each preceded by Sagittal notation, in short-ASCII form, for the pitch that will make that interval with a C below it. "\" may be pronounced "down" and "/" may be pronounced "up". In long-ASCII form these are \! and /|. In a proper Sagittal font they appear as half arrows with a left barb only, down and up.



22edo Interval names

The same information is presented in table form below.

22edo interval names

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 0,   0c, C   P1, Db\ d2
 1,  55c, C/  S1, Db sm2
 2, 109c, C#\ A1, Db/ m2
 3, 164c, C# SA1, D\  M2
 4, 218c, Eb\ d3, D  SM2
 5, 273c, Eb sm3, D/  A2
 6, 327c, Eb/ m3, Fb sd4
 7, 382c, E\  M3, Fb/ d4
 8, 436c, E  SM3, F\  s4
 9, 491c, E/  A3, F   P4
10, 545c, Gb sd5, F/  S4
11, 600c, Gb/ d5, F#\ A4
12, 655c, G\  s5, F# SA4
13, 709c, G   P5, Ab\ d6
14, 764c, G/  S5, Ab sm6
15, 818c, G#\ A5, Ab/ m6
16, 873c, G# SA5, A\  M6
17, 927c, Bb\ d7, A  SM6
18, 982c, Bb sm7, A\  A6
19,1036c, Bb/ m7, Cb sd8
20,1091c, B\  M7, Cb/ d8
21,1145c, B  SM7, C\  s8
22,1200c, B/  A7, C   P8
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Defending the thesis

It is an important feature of this system that, within any given EDO, there is a one-to-one mapping between accidentals in the pitch notation and qualifiers in the interval naming. For example:

For all 22edo interval classes that admit of "minor" and "major" qualifiers we have:

b\ diminished
b   subminor
b/ minor
\   major
    supermajor
/ augmented

For all 22edo interval classes that admit of "perfect" qualifiers we have:

b   subdiminished
b/ diminished
\   sub
    perfect
/   super
#\ augmented
superaugmented

This is of course different from their mapping in 12edo, but there is no reason why this mapping must remain constant across all temperaments. Different mappings are what different temperaments are all about. The very fact that the seconds, thirds, sixths and sevenths with no accidental are no longer major but supermajor, and the unisons, fourths and fifths with a flat and sharp are no longer diminished and augmented but subdiminished and superaugmented, tells us that we are not in meantone Kansas anymore. It tells us that we are over the rainbow in superpythagorean land.

It may seem that I'm being inconsistent in (a) promoting EDO pitch notations where the 7 nominals are in a *single* chain of fifths (having codeveloped Sagittal notation) and (b) promoting an interval naming system where the 7 middle-of-class intervals are in *two* chains of fifths a half-fifth apart. But in fact the two systems have very different requirements. The fact that a 7-note linear scale has (in general) 13 different intervals makes it clear that pitches and intervals are very different things.

A pitch notation needs to deal with modulation, so it needs to be very uniform indeed. We have the example of Johnston notation whose nominals are based on the JI major scale. It is a complete nightmare under modulation. The nominals of a pitch notation must be linear, and if we are to maintain any connection with standard pitch notation, the generator must be a fifth.

An interval naming scheme on the other hand is completely unaffected by modulation. When the pitches modulate, the intervals stay exactly the same. And the conventional naming system already has two superclasses of intervals - the perfects and the major/minors. By putting them into two different chains of fifths we end up with chains that are so short that the size of the intervals at the extremes is not so greatly affected by the change in the size of the fifths. The thirds and sixths change only half as much as the fifths do, and the seconds and sevenths only one and a half times as much. This allows us to keep close to the conventional sizes for all the interval names.

Some other EDOs

Below, I give interval names obtained by this method for some other EDOs, in an abbreviated form. I have omitted alternative names for most intervals, but when I have included them, they are joined by a slash.

5 edo P1 A2/d3 P4 P5 A6/d7
7 edo P1 N2 N3 P4 P5 N6 N7
11 edo P1 sm2 N2 N3 SM3 P4 P5 sm6 N6 N7 SM7 *
13 edo P1 N2 SM2 sm3 N3 P4 A4 d5 P5 N6 SM6 sm7 N7 *
15 edo P1 m2 M2 A2/d3 m3 M3 P4 A4 d5 P5 m6 M6 A6/d7 m7 M7
16 edo P1 A1/d2 m2 M2/d3 m3 M3 A3/d4 P4 A4/d5 P5 A5/d6 m6 M6 A6/m7 M7 A7/s8
17 edo P1 A1/sm2 N2 SM2/sd3 sm3 N3 SM3/d4 P4 A4/dd5 AA4/d5 P5 A5/sm6 N6 SM6 sm7 N7 SM7
19 edo P1 sm2 m2 M2 SM2sm3 m3 M3 SM3/s4 P4 S4/sd5 SA4/s5 P5 s5/sm6 m6 M6 SM6/sm7 m7 M7 SM7
21 edo P1 S1/sm2 m2 N2 M2/sm3 m3 N3 M3 s4/SM3 P4 S4/d5 s5/A4 P5S5/sm6 m6 N6 M6/sm7 m7 N7 M7 SM7
22 edo P1 sm2 m2 M2 SM2 sm3 m3 M3 SM3 P4 S4 A4/d5 s5 P5 sm6 m6 M6 SM6 sm7 m7 M7 SM7
23 edo P1 S1 sm2 m2 M2/d3 sm3 m3 M3 SM3/d4 s4/A3 P4 S4 s5 P5 S5 A5/sm6 m6 M6 sm7 m7 M7 SM7 s8
26 edo P1 S1 sm2 m2 M2 SM2/d3 sm3 m3 M3 SM3/d4 s4/A3 P4 S4 A4/d5 s5 P5 S5 sm6 m6 M6 SM6 sm7 m7 M7 SM7 s8
41 edo P1 W1 d2 nm2 m2 N2 M2 WM2 A2 sA2/sm3 nm3 m3 N3 M3 WM3/d4 s4 n4 P4 W4 S4 A4 d5

* The interval names given above for **11edo** and **13edo** are included to show how the system copes with such extreme fifth sizes, but it would make more sense to name their intervals as subsets of 22edo and 26edo (or 39edo) respectively.

Why 72?

Why use 72edo as the basis of this nomenclature? Because, outside of the effects of particular melodic scales, the most salient points on the spectrum of dyads are the justly-intoned dyads -- those that correspond to small whole-number ratios of frequency. 72edo approximates many of these with uncanny precision and consistency, while bringing all the advantages of a uniformly spaced system.

It is not necessary to know anything about frequency ratios, or just intonation, to use this system. However, for those who are interested, I provide the following table.

72edo degree	Interval name	Ratio
0	P1	1:1
7	m2	15:16
9	N2	11:12
10	WN2	10:11
11	M2	9:10
12	WM2	8:9
14	SM2	7:8
16	sm3	6:7
18	nm3	27:32
19	m3	5:6
21	N3	9:11
22	WN3	13:16
23	M3	4:5
25	nSM3	11:14
26	SM3	7:9
30	P4	3:4
33	S4	8:11
35	A4	5:7
37	d5	7:10
39	s5	11:16
42	P5	2:3
46	sm6	9:14
45	Wsm6	7:11
49	m6	5:8
50	nN6	8:13
51	N6	11:18
53	M6	3:5
54	WM6	16:27
56	SM6	7:12
58	sm7	4:7
60	nm7	9:16
61	m7	5:9
62	nN7	11:20
63	N7	6:11
65	M7	8:15
72	P8	1:2

This also helps to explain how it was decided which degrees of 72 would have the shorter names (without narrow or wide). In general the simpler ratio gets the simpler name, but there are some exceptions due to the fact that we want symmetry between intervals and their octave-inversions, as well as wanting seconds to agree with thirds (as inversions within a tetrachord). Any symmetry that can be built into the system reduces the cognitive load on the user.

Further information on the origin of this terminology can be found in my earlier paper A note on the naming of musical intervals.

-- Dave Keenan, 2015-Aug-27

Updated 2016-Jun-01, to correct some 72edo degree numbers in the final table and add a note that "small" and "Large" may be substituted for "narrow" and "Wide".